Unit V: Mechanical Energy

Work

In physics, we have two definitions of work.

1) Work is a transfer of energy.

This means that energy changes forms or energy is transferred from one object to another object. When a transformation takes place, not all of the energy is used to produce useful work. Some is converted into heat or other types of energy. For now, just know that energy is the ability to do work.

2) Work happens when a force causes an object to move through a displacement.

If the force exerted on an object does not cause a displacement, no work is done. While causing a displacement, the force must also be in the same direction as the displacement.

The unit of work is the Nm. This is commonly called the Joule (J). Work is a scalar quantity, but comes from the product of two vectors. Positive work is done when the applied force and the displacement are in the same direction. Negative work is done when they are in opposite directions.

Ex. Holding a pizza in your hands, you walk 8 m to the other side of the room. Did you do any work?

No work. Force is perpendicular to displacement.

Ex. Wanting to get stronger for next season, you go to the weight room to better yourself. If you lift 75 kg from the ground, over your head, and back down to the floor, have you done any work?

No. No displacement.

Ex. After getting his Porsche fixed, Mr. Birrell (again) gets stuck in a snowbank. He promises a passing student a passing grade if they do some work. Both people get behind the car and push it out of the snow. Did the student do work?

Yes. Force was in direction of displacement.

Ex. After getting tired of holding your 15 kg Physics binder, you put it on the floor. Did you do any work?

- no if you drop it (gravity did the work)

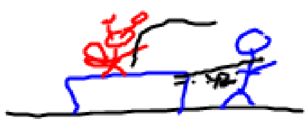
- yes if moved not entirely by gravity (negative work)

Both definitions of work lead to equations:

$W = \Delta E$	
$W = F\Delta d cos\theta$	Where W = work (Joules) (J) F = force (N) Δd = displacement (m) Θ = angle between the force and displacement vectors

There is an angle in the second formula because it is possible for work to be done when the vectors of force and displacement are not parallel. The only time when no work is said to be done is when the forces are perfectly perpendicular to one another. The cosine θ calculates the component of the force in the direction of the displacement. So, if the force and displacement are perpendicular, the cosine of 90° is 0 and leads to no work.

Ex. Pulling your new insect overlord's sled at an angle of 42°.



Work is done as part of force is in direction of Δd

Ex. You pull the sled horizontally with a force of 385 N for 48 m. Find the work done.

$F = 385N$ $\quad W = F\Delta d \cos\theta$

$\Delta d = 48m$ $\quad W = 385N(48m)\cos 0°$

$W = ?$ $\quad W = 18480J$ $\quad \boxed{W = 18000J}$

$\theta = 0°$

Ex. You pull the sled at an angle of 42° with a force of 385 N for 48 m until your insect overlord eats you. Determine the amount of work done before going to your painful death.

$F = 385N$ $\quad W = F\Delta d \cos\theta$

$\Delta d = 48m$ $\quad W = 385N(48m)\cos 42°$

$\theta = 42°$ $\quad W = 13733J$

$W = ?$ $\quad \boxed{W \approx 14000J}$

Kinetic Energy

Energy is a scalar quantity measured in Joules. There are many types of energy; we will look at but three types.

Any object that is moving has kinetic energy. Like momentum, we can calculate how much kinetic energy an object possesses.

$E_k = \frac{1}{2}mv^2$	Where E_k = kinetic energy (J) m = mass (kg) v = speed (m/s)

We define any object as anything that has a mass. Also keep in mind that motion is relative. An object may have different amounts of kinetic energy depending upon its frame of reference. Usually, the surface of the Earth will be our reference point.

Ex. A distraught chicken is falling off a roof of a barn. How much kinetic energy does it have relative to its intestinal parasite?

0 J → not moving relative to the parasite in its own stomach

How much kinetic energy does it have relative to someone standing on the ground if it is moving at 6.5 m/s?

$m = 3.4\,kg$
$v = 6.5\,m/s$
$E_k = ?$

$E_k = \frac{1}{2}mv^2$
$E_k = \frac{1}{2}(3.4\,kg)(6.5\,m/s)^2$
$E_k = 71.825\,kg\,m^2/s^2$

$\boxed{E_k \approx 72\,J}$

Ex. Determine the speed of the 3.4kg chicken if it has 551J of kinetic energy.

$E_k = 551\,J$
$m = 3.4\,kg$
$v = ?$

$E_k = \frac{1}{2}mv^2$
$2E_k = mv^2$
$\frac{2E_k}{m} = v^2$
$\pm\sqrt{\frac{2E_k}{m}} = v$

$v = \sqrt{\frac{2(551\,J)}{3.4\,kg}}$

$\boxed{v = 18\,m/s}$

Ex. Driving his Porsche down the road at 124 km/h, Mr. Birrell notices a school zone ahead. He hits the brakes and slows down to 37 km/h. If he slowed down over 39.6 m, find the average force applied by the brakes. $m = 1500\,kg$

$v_1 = 124\,km/h = 34.4\,m/s$
$v_2 = 37\,km/h = 10.27\,m/s$
$\Delta d = 39.6\,m$
$F = ?$
$m = 1500\,kg$
$\theta = 180$

$W = \Delta E$
$W = F\Delta d\cos\theta$
$\Delta E = F\Delta d\cos\theta$
$E_{k2} - E_{k1} = F\Delta d\cos\theta$
$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = F\Delta d\cos\theta$

$F = \frac{\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2}{\Delta d\cos\theta}$

$F = \frac{\frac{1}{2}(1500)(10.27)^2 - \frac{1}{2}(1500)(34.4)^2}{39.6\,m\cos 180}$

$F = 20414.53\,N$

$\boxed{F \approx 2.0\times10^4\,N}$

The concept that work is equal to the change in energy is called the Work-Energy Theorem. This means that doing work on an object changes the amount of mechanical energy it has. Mechanical energy is not just kinetic energy – the objects potential energy may change as well.

Potential Energy

Potential energy is stored energy that is able to do work later. There are many types.

Ex. Batteries- chemical

Stretched Elastic- elastic /spring

Matter- "rest energy" $E = mc^2$

Holding a rock above your head- Gravitational

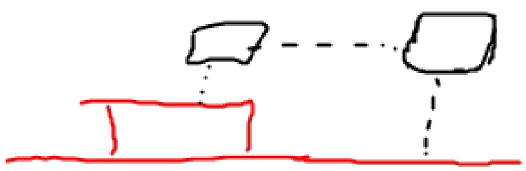
Gravitational Potential Energy

Gravitational potential energy is energy stored as a result of the position of an object relative to ground level or some other arbitrary reference called a base level.

$E_p = mg\Delta h$	Where E_p = gravitational potential energy m = mass of object being raised g = acceleration due to gravity (m/s^2) Δh = height (m) object is over base level

Bizarrely, you can change the amount of potential energy an object has by moving it horizontally.

Ex.



Ex. Determine the gravitational potential energy of a 3.4 kg dog 26.7 m above the ground.

$E_p = ?$
$m = 3.4 kg$
$\Delta h = 26.7 m$

$E_p = mg\Delta h$

$E_p = 3.4 kg(9.81 m/s^2)(26.7 m)$

$\boxed{E_p \approx 890 J}$

Ex. Determine the gravitational potential energy of the same dog at the same height using their owner's outstretched arms at 1.4 m above the ground as the base level.

$\Delta h = 26.7 m - 1.4 m = 25.3 m$

$E_p = 3.4 kg(9.81 m/s^2)(25.3 m)$

$\boxed{E_p \approx 840 J}$

Which is the correct answer for how much potential energy the dog has?

Both for their reference points.

Ex. Determine the height of an 85 kg skydiver if they have 2.08 x 10^7 J of potential energy.

$\Delta h = ?$
$m = 85 kg$
$E_p = 2.08 \times 10^7 J$

$E_p = mg\Delta h$

$\frac{E_p}{mg} = \Delta h$

$\Delta h = \frac{2.08 \times 10^7 J}{85 kg(9.81 m/s^2)}$

$\boxed{\Delta h \approx 25000 m}$

Elastic Potential Energy

Anything that can have its shape changed and then return to its original shape can store elastic potential energy.

Robert Hooke, a man who did so many things for science, came up with Hooke's Law:

$F_s = -kx$	Where Fs = force (N) k = spring constant (N/m) x = amount of expansion (+) or compression (-) (m)

The minus sign in the formula above means that the formula calculates the amount of force trying to restore the spring back to its original position.

Ex. A spring has a spring constant of 20.3 N/m. Find the restoring force when:

a) The spring is extended 12 cm.

$\rightarrow x$
$x = 12 cm = 0.12 m$
$k = 20.3 N/m$
$F_s = ?$

$F_s = -kx$

$F_s = -20.3 N/m(0.12m)$

$F_s = -2.436 N$

$\boxed{F_s \approx -2.4N}$

b) The spring is compressed 21 cm from its starting point.

$\rightarrow x$
$x = -21 cm = -0.21 m$
$k = 20.3 N/m$
$F_s = ?$

$F_s = -kx$

$F_s = -20.3 N/m(-0.21m)$

$F_s = 4.263 N$

$\boxed{F_s \approx 4.3N}$

The above formula can be used to find a formula for elastic potential energy:

$$E_p = \frac{1}{2}kx^2$$

Ex. Find how much energy a composite hockey stick can store if it has a spring constant of 11000 N/m and it is flexed 0.12 m.

$k = 11000 N/m$
$x = 0.12 m$
$E_p = ?$

$E_p = \frac{1}{2}kx^2$

$E_p = \frac{1}{2}(11000 N/m)(0.12 m)^2$

$E_p = 79.2 J$

$\boxed{E_p \approx 79 J}$

Conservation of Energy

Often, it is useful to look at the total mechanical energy of a system. While there is no formal formula for total mechanical energy, it could look something like this:

$$E_m = E_k + E_p$$

Ex. A UFO is cruising at an altitude of 867 m while looking for a dog to kidnap. If it is travelling at a constant speed of 324 km/h, find the total mechanical energy of the UFO. The UFO mass is 2500kg.

$\Delta h = 867m$
$v = 324 km/h = 90.0 m/s$
$m = 2500kg$

$E_m = E_k + E_p$
$E_m = \frac{1}{2}mv^2 + mg\Delta h$
$E_m = \frac{1}{2}(2500kg)(90.0 m/s)^2 + 2500kg(9.81 m/s^2)(867m)$
$E_m = 31388175 J \approx \boxed{3.1 \times 10^7 J}$

Recall that an isolated system means that no matter or energy can enter or leave. This means that you end with what you start with. No external forces are acting on the system.

If we have an isolated system, then energy is conserved. Total mechanical energy before will equal the total mechanical energy after.

$$E_m = E'_m$$

$$E_k + E_p = E'_k + E'_p$$

This is similar to the Law of Conservation of Momentum. In a twist of events that may shock you, this law is called the Law of Conservation of Energy. If one type of energy decreases, the other type will have to increase to balance everything out.

Ex. On a cold winter night, you find yourself atop a 35.8 m hill with your trusty GT Racer. If you and the GT have a combined mass of 81 kg, how fast will you be going at the bottom of the hill? Assume there is no friction.

$m = 81kg$
$\Delta h = 35.8m$
$\Delta h' = 0m$
$v = 0 m/s$
$v' = ?$

$E_m = E_m'$
$E_p = E_k'$
$\cancel{m}g\Delta h = \frac{1}{2}\cancel{m}v'^2$
$g\Delta h = \frac{1}{2}v'^2$
$2g\Delta h = v'^2$
$\pm\sqrt{2g\Delta h} = v'$
$\sqrt{2(9.81 m/s^2)(35.8m)} = v'$
$v' = 26.503 m/s$
$\boxed{v' \approx 27 m/s}$

Ex. In an attempt to discover the physics laws of our universe, the aforementioned UFO will be dropping a kidnapped dog from a height of 265 m. Find the velocity of the dog at the following heights above the ground, assuming no air resistance: The mass of the dog is irrelevant!

a) 265 m

0 m/s → has all potential energy

b) 142 m

$\Delta h = 265m$

$\Delta h' = 142m$

$v = 0m/s$

$v' = ?$

$E_m = E_m'$

$\cancel{E_k} + E_p = E_k' + E_p'$

$\cancel{m}g\Delta h = \frac{1}{2}\cancel{m}v'^2 + \cancel{m}g\Delta h'$

$g\Delta h = \frac{1}{2}v'^2 + g\Delta h'$

$g\Delta h - g\Delta h' = \frac{1}{2}v'^2$

$2(g\Delta h - g\Delta h') = v'^2$

$\pm\sqrt{2(g\Delta h - g\Delta h')} = v'$

$\sqrt{2[(9.81)(265m) - (9.81)(142m)]} = v'$

$\boxed{v' = 49 \text{ m/s}}$

c) 0m

$E_m = E_m'$

$E_p = E_k$

$\cancel{m}gh = \frac{1}{2}\cancel{m}v'^2$

$\sqrt{2gh} = v'$

$v' = \sqrt{2(9.8) \text{ m/s}^2 (265m)}$

$\boxed{v' = 72 \text{ m/s}}$

Ex. A toy car with mass of 212g is pushed by a student along a track so that it is moving at 12 m/s. It hits a spring (k = 52.8 N/m) at the end of the track, causing it to compress.

a) Determine how far the spring compressed to bring the car to a stop.

$m = 0.212kg$

$v = 12 m/s$

$k = 52.8 N/m$

$x' = ?$

$E_m = E_m'$

$E_k + \cancel{E_p}^0 = \cancel{E_k'}^0 + E_p'$

$\cancel{\frac{1}{2}}mv^2 = \cancel{\frac{1}{2}}kx'^2$

$\pm\sqrt{\frac{mv^2}{k}} = x'$

$x' = \sqrt{\frac{0.212kg(12 m/s)^2}{52.8 N/m}}$

$\boxed{x' = 0.76m}$

b) If the spring only compressed 50 cm in bringing the car to a stop, explain what happened.

- the missing energy most likely converted to waste heat

Power

Power is the rate at which work is done. It is a measure of how quickly energy is being used. Power is measured in J/s, which is also called the Watt (W).

$P = \frac{W}{t} = \frac{\Delta E}{t}$	Where P = power (Watts) (J/s) W = ΔE = work (Joules) t = time (seconds)

It is important to note that electricity is not power – electricity is energy.

Ex. If you leave a 150W bulb on for 8.5 h, determine how much electricity you have used.

$\Delta E = ?$
$P = 150W$
$t = 8.5h$
$= 30600s$

$P = \frac{\Delta E}{t}$
$Pt = \Delta E$

$\Delta E = 150W(30600s)$
$\Delta E = 4590000J$
$\boxed{\Delta E \simeq 4.6 \times 10^6 J}$

Many of the questions we look at involved motors lifting something at a constant velocity. A higher power motor will be able to lift the object faster. When the object being lifted is moving at a constant velocity, we can use the following:

$P = \frac{W}{t}$ $P = \frac{F\Delta d}{t}$ $P = Fv$	Where P = power (W) F = force (N) v = average velocity (m/s)

Ex. A 220 W motor is being used to lift plywood at a constant speed onto a roof. If one sheet of plywood has a mass of 34 kg, find the speed at which the plywood will be raised at.

$P = 220W$
$m = 34Kg$
$v = ?$
$F = F_g = mg$

$P = Fv$
$P = mgv$
$\frac{P}{mg} = v$

$v = \frac{220W}{9.81 m/s^2 (34Kg)}$
$\boxed{v \simeq 0.66 m/s}$

Efficiency

Efficiency can be measured in terms of either energy or power. It is a comparison of how much useful stuff comes out (output) compared to what was originally put in (input).

There is no such thing as a 100% efficient device – the output will always be less than the input. In any conversion, there is some waste energy that is turned into forms you don't necessarily want.

$efficiency = \frac{\text{Energy out}}{\text{Energy in}} = \frac{E_o}{E_I}$ or $\frac{\text{Power out}}{\text{Power In}} = \frac{P_o}{P_I}$

Ex. An incandescent bulb has an efficiency of only 2.0%. Find the amount of power that is turned into light. 60W

efficiency = 2.0% = 0.020

$P_I = 60W$

$P_o = ?$

$\text{efficiency} = \frac{P_o}{P_I}$

$P_I(\text{efficiency}) = P_o$

$P_o = 60W(0.02)$

$\boxed{P_o = 1.2W}$

Ex. A crane is lifting a 380 kg load at a constant velocity of 3.2 m/s. Determine its efficiency if the motor on the crane is rated at 22 000 W.

$m = 380kg$

$v = 3.2 m/s$

efficiency = ?

$P_I = 22000W$

$P_o = ?$

$P_o = Fv$

$P_o = mgv$

$P_o = 380kg(9.81 m/s^2)(3.2 m/s)$

$P_o = 11928.96W$

$\text{efficiency} = \frac{P_o}{P_I}$

$= \frac{11928.96W}{22000W}$

$\approx 0.54 \approx 54\%$

How many horsepower is the engine in the above example?

1 hp = 746W

$22000\cancel{W}\left(\frac{1 hp}{746\cancel{W}}\right) = 29 hp$